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14. ABSTRACT It can be shown that ion beams in electric propulsion devices create a potential well which acts to retain neutralizing electrons to a degree. To be trapped in the well, however, it is necessary for the energy and momentum that got the electrons to the well to "bounce" off the other side of the well to bring their dwell time up to a point where they can be trapped. We demonstrate that conditions exist in normal electric propulsion plumes where a collisional scattering mechanism can be sufficient to scatter a neutralizing electron beam into the ion beam. Furthermore, once in the well for a sufficiently long period, collective instabilities such as the Buneman instability thermalize the electrons, dropping the bulk electron velocity to match that of the ions. While normally this would mean that electron temperature should be equal to the well depth, we show by means of a simple flux model that electrons thermalize only to a point where the flux of "hot" electrons out of the well is matched by the ambient "cold" electrons moving into the well.					
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Collisional Scattering Into and Evaporative Cooling From a Potential Well

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Nomenclature

b, b_0	=	impact parameter, reference impact parameter
bg	=	subscript to designate background quantity
m_e, m_i, m_r	=	electron mass, ion mass, reduced mass
n	=	number density
q	=	particle charge
T	=	temperature
u	=	relative velocity
v_{\min}, v_{avg}	=	minimum velocity for instability growth or well escape, average velocity
γ	=	growth rate
δf	=	well depth to well temperature ratio
θ	=	injection angle to beam
$\ln \Lambda$	=	Coulomb logarithm
ν, ν_{eff}	=	collision frequency, effective collision frequency
τ, τ_{scat}	=	slowdown time, slowdown time for scattering into potential well
χ, χ_{scat}	=	deflection angle, minimum deflection angle for scattering into potential well

I. Introduction

In order to have proper functioning of an electric propulsion thruster, it is necessary to provide a neutralizing source of electrons to mitigate both the spacecraft charging due to the ion current and the space charge of the plume itself. While the dual requirements have been empirically understood for some time,¹ the mechanism in achieving them has not been clearly explained, despite extensive theoretical work through the years.^{2,3,4,5,6 (among others)} Computer simulations have been similarly unrevealing.^{7,8,9,10,11} What has emerged is a complex picture where multiple processes may be at work, from electrostatic trapping to collective instabilities to conventional Coulomb

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collisions. The blend of processes has confounded efforts to produce a single, coherent picture of the neutralization process.

The authors have previously examined the potential role of Coulomb collisions and instabilities.¹² Even with a detailed treatment including larger-angle collisions, it has been shown that Coulomb collisions are too weak to perform the neutralization function alone^{4,12}. The Buneman instability¹³ appears to be a strong candidate, but there is still a question of the electron velocity transverse to the beam resulting in an insufficient dwell time to allow the instability growth. There are also measurements from the Deep Space 1 (DS1) mission as reported by Wang et al.,¹⁴ which show that the electron temperature in electron volts in the thruster plume is lower by a factor of about seven than the well depth. This is significant because the Buneman instability can be shown to heat the electrons to a temperature approximately equal to the well depth. At first glance, this would rule out instabilities to generate the turbulent fluctuations needed for rapid neutralization and force another look at Coulomb collisions, but for one important point: Beyond the initial few seconds, the electrons moving into the beam is not just due to the neutralizer on the spacecraft, but rather exchanges from the ambient plasma background. Hot electrons can escape from the beam and be replaced with cooler electrons from the ambient environment. This “evaporative cooling” effect allows the trapped electrons to maintain a cooler temperature and will be shown to roughly match the observations on DS1 with a simple flux calculation. While this does not explain the ability of a neutralizer to function with a closely coupled current balance, it does explain how observations can show such cool electron temperatures.

With a possible explanation of the cool electrons, there is only the question of initial transverse velocity as a potential problem with conventional methods adequately describing neutralization. We propose that with a small adjustment to basic Coulomb collision theory it is possible to show sufficient scattering for electrons to not have an exit vector even though they are energetically capable of escaping. Once in the well, electrons will match their bulk velocity with the ions through conventional Coulomb collisions or collective instabilities.

II. Smaller-Angle Scattering Through Coulomb Collisions

The classic Rutherford scattering equation describes the relationship between the scattering angle χ and the electrostatic force between the two particles, their relative velocity u , their reduced mass m_r , and the impact parameter b . Traditionally written

$$\tan \frac{\chi}{2} = \frac{qq_{bg}}{4\pi\epsilon_0 m_r u^2 b}, \quad (1)$$

the reference impact parameter to achieve scattering in a single collision through 90° is called b_0 and we can write (1) as

$$\tan \frac{\chi}{2} = \frac{b_0}{b}. \quad (2)$$

Continuing the derivation to include the collective effects of numerous small angle collisions introduces the Coulomb logarithm $\ln \Lambda$, and we get the classic definition of a slowdown time τ or collision frequency ν of

$$\tau = \frac{4\pi\epsilon_0^2 m_r^2 u^3}{n_{bg} (qq_{bg})^2 \ln \Lambda} = \frac{1}{\nu}. \quad (3)$$

If we redefine the reference impact parameter to instead be sufficient to achieve some minimum scattering χ_{smin} , the slowdown time will instead be an e-folding time to achieve collective scattering of χ_{smin} . Solving (1) with χ_{smin}

$$b = \frac{qq_{bg}}{4\pi\epsilon_0 m_r u^2} \left(\tan \left(\frac{\chi_{smin}}{2} \right) \right)^{-1}. \quad (4)$$

Using the same definition of b_0 for 90° scattering, this makes (2) now

$$\tan \frac{\chi}{2} = \frac{b_0}{b} \left(\tan \left(\frac{\chi_{smin}}{2} \right) \right)^{-1}. \quad (5)$$

As χ_{smin} is a constant, it flows through the rest of the derivation untouched and we can write a new τ as

$$\tau_{smin} = \frac{4\pi\epsilon_0^2 m_r^2 u^3}{n_{bg} (qq_{bg})^2 \ln \Lambda} \tan^2 \left(\frac{\chi_{smin}}{2} \right). \quad (6)$$

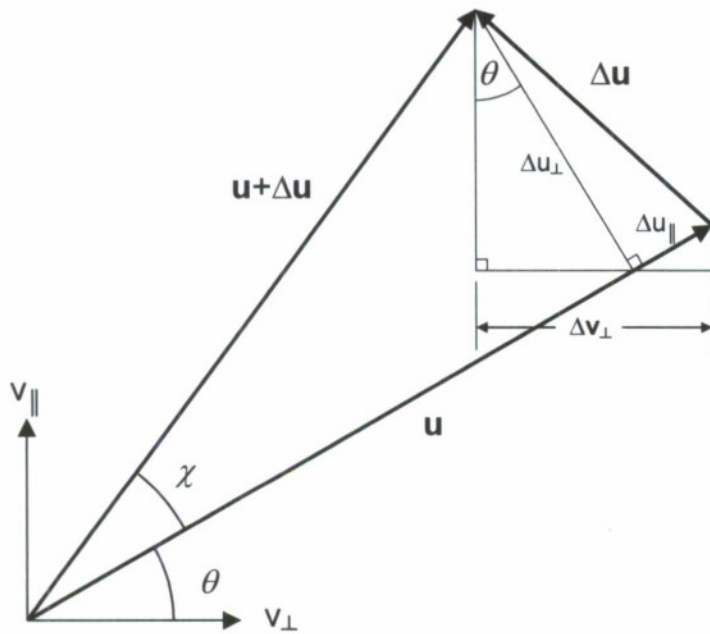


Figure 1: Incidence angle θ , deflection angle χ and velocity change vector $\Delta \mathbf{v}_\perp$.

$$\Delta \mathbf{v}_\perp = \Delta u_\parallel \cos \theta + \Delta u_\perp \sin \theta \quad (7)$$

we can solve for the required χ to get an acceptable Δv_\perp in motion perpendicular to the beam. We also note that χ can be negative – the deflection can be in the opposite direction, scattering more perpendicular to the beam. However, assuming a positive θ , the total deflection across all χ still shows bias in the direction of the beam. This can be seen in Figure 2.

By setting the velocity perpendicular to the beam equal to (7), we can get

$$v_\perp = v \sin \chi_{s \min} \sin \theta + 2v \sin^2 \frac{\chi_{s \min}}{2} \cos \theta \quad (8)$$

Since $\mathbf{v}_\perp = \mathbf{v} \cos \theta$, we can drop out the velocity magnitudes and solve for χ as a function of θ . Only one physical root emerges from (8):

$$\chi_{s \min} = 2 \arccos \left(\frac{\sqrt{1 + \sqrt{\sin^2 \theta}}}{\sqrt{2}} \right). \quad (9)$$

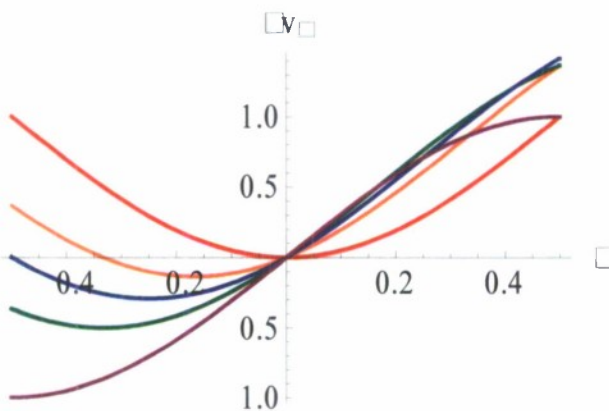


Figure 2: Δv vs. χ for incidence angle of 0(red), 15,(orange), 30(blue), 45(green), 90(purple) degrees.

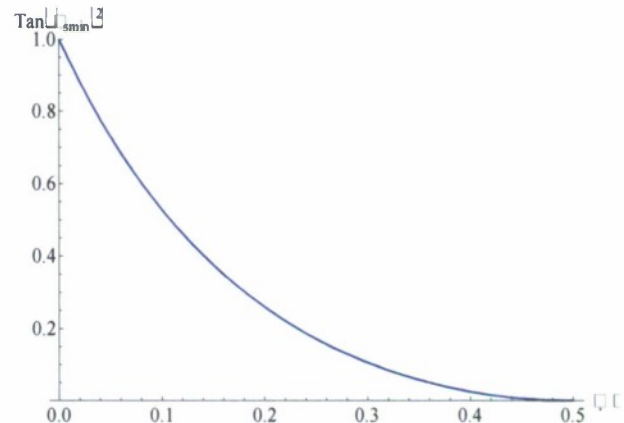


Figure 3: $\tan^2(\chi_{s \min}/2)$ vs. incidence angle.

At this point, it is necessary to solve for $\chi_{s \min}$. Since the expected interaction times are small, we will first assume that there is no energy loss during the period of scattering. This should be justified because the energy loss time constant is greater than the momentum loss time constant by roughly $m_i / 2m_e$.¹⁵ Because of this, both the pre- and post-scattering vectors have the same magnitude. This leads us to the picture in Figure 1. Standard Coulomb collision theory solves for the deflections Δu_\parallel and Δu_\perp , so by combining them as

Inserting this into $\tan^2(\chi_{s\min}/2)$, we get a modifier for the slowdown time, which is plotted in Figure 3. As expected by intuition, changing the incidence angle changes the necessary deflection time from 100% of the normal slowdown time at perfectly perpendicular to the beam to 0 at perfectly aligned with the beam and therefore having no perpendicular velocity to lose.

To determine what the distance required to achieve a scattering through $\chi_{s\min}$, we use the equation

$$\Delta x = v_0 t \cos \theta [1 - \tau \exp(-t/\tau)] \quad (10)$$

Setting $t = 3\tau$, with a density of $n=1e15 \text{ m}^{-3}$, the results are plotted in Figure 4. We can see that while a perfectly perpendicular electron requires an energy of less than 1eV to slow sufficiently for a 1m beam, one with an angle 45 degrees off perpendicular is capable of deflecting electrons over to 4eV in energy. Considering that neutralization electrons typically are generated from hollow cathodes, they will likely have a few eV of energy to start with, and potentially a few more from any potential difference between the cathode and the beam. Breida,¹⁶ for example, uses a 5V potential difference between the beam and the cathode, with a 1eV temperature. The objective is not to entirely slow the electrons, but slow them just enough that they are trapped long enough for other effects to match bulk velocities with the ions.

We have now shown that collisional scattering, while likely insufficient to totally slow electrons within a reasonable timeframe, can be sufficient to scatter them into the well where further collisions or instabilities can match bulk velocities so ion and electron currents match.

III. Velocity Matching Through Instabilities

Although it was shown that collisions can scatter electrons into the ion beam, ultimately they must match bulk velocities if a sustained current neutralization is to be maintained. For relatively cold electrons moving rapidly with respect to the ions, collective instabilities can easily be a driver. For electrons moving rapidly against a mobile ion background, the Buneman instability¹³ is the correct model. Ishihara, Hirose, and Langdon^{17,18} have solved for the maximum growth rate as

$$\gamma = \frac{\sqrt{3}}{2} \left(\frac{m_e}{2m_i} \right)^{\frac{1}{3}} \left[1 + \frac{1}{2} \left(\frac{m_e}{2m_i} \right)^{\frac{1}{3}} \right] \omega_{pe} \quad (11)$$

with an effective collision frequency during the growth of the instability of

$$\nu_{eff} \approx 0.53 \left(m_e/m_i \right)^{0.61} \omega_{pe}. \quad (12)$$

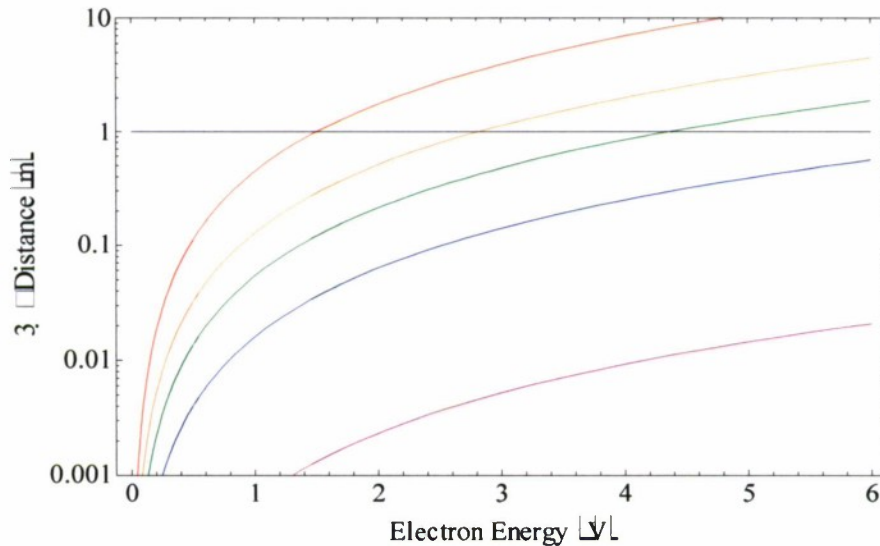


Figure 4: Distance traveled in 3τ vs. initial energy with angle from perpendicular of 0 (red), 15 (orange), 30 (green), 45 (blue), and 80 (purple) degrees. Reference line at 1m in black.

For $n=10^{15} \text{ m}^{-3}$ and a xenon beam, the effective collision frequency is $2.77e-4\omega_{pe} = 4.95e5 \text{ s}^{-1}$. This is significantly smaller than the effective collision frequency used by Parks¹⁰, but is somewhat higher than Coulomb collision frequencies. Using (10), we can examine the distance that an electron will travel using v_{eff} as given by (12). Using the same parameters, the Buneman instability should have a completely thermalized beam within 37.35cm of the entrance point. While that is a distance large enough to have varying beam parameters, it is still small in relation to the diameter of the beam, suggesting that electrons should be fairly well randomized if they arrive in a streaming fashion. Similarly, if they are streaming along the beam, it will provide a thermal spread of electrons somewhat more rapidly than Coulomb collisions alone.

To trigger the Buneman instability, the electrons must be traveling relative to the ions at a velocity greater than

$$v_{min} = 0.926(2kT / m_e)^{1/2} \left[1 + (m_e / m_i)^{1/2} \right]. \quad (13)$$

This is known as the Buneman Critical Velocity.¹³ Since the electron/ion mass ratio is very small, the critical velocity can be thought of as approximately the thermal velocity. When the instability has thermalized the plasma sufficiently that the critical velocity condition no longer holds, the instability has saturated and other effects, such as the ion acoustic instability or classic Coulomb collisions, take over to match velocities and temperatures. The shutdown mechanism also suggests that electrons will thermalize to completely fill the potential well, i.e. $\phi_{well} \sim T_e$. That electron temperatures of this level are not observed¹⁴ is problematic, but will be discussed below.

For a typical beam injection as discussed above, the electrons are moving at a few eV at the bottom of the well, with a temperature of about 1-2 eV. This suggests that the Buneman instability will be crucial in thermalizing the directed motion from the well to a point where electrons will remain within it.

IV. "Evaporative" Cooling

The question of temperature in the beam remaining lower than the well depth can be explained by one simple concept: evaporation. The high energy tail of the electron distribution escapes until the well deepens enough to retain the electrons. Meanwhile, the ambient background is supplying cool electrons to replace those which escaped. This substitution of low-energy electrons for high-energy ones can create an appearance of a cooler electron distribution than may otherwise be.

While an NSTAR-class ion engine can produce about a 1A beam, eventually the current from the background will become much greater. Figure 5 shows the current into the beam vs. time for a 1m diameter 1kV xenon beam for various background densities. This shows that we cannot rely on the beam filling effect to achieve neutralization initially, but eventually the conditions of the background environment will dwarf whatever the neutralizer is providing.

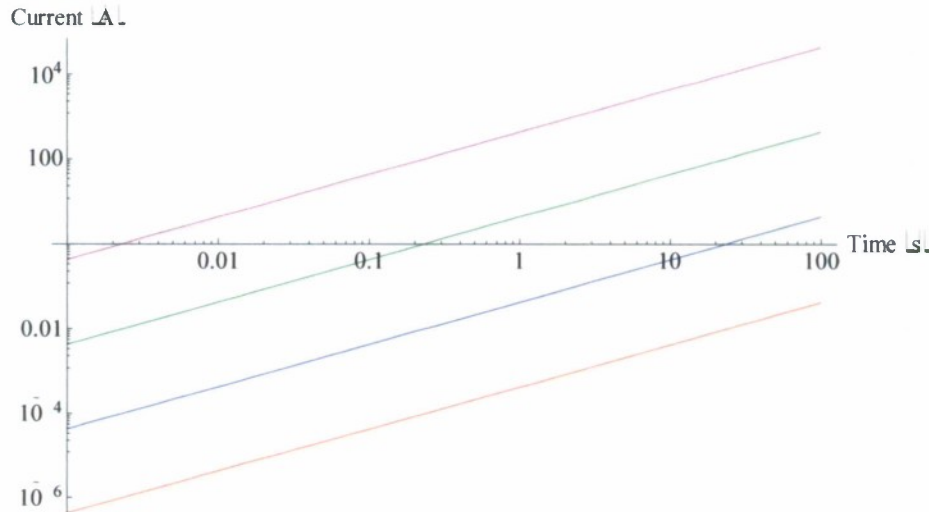


Figure 5: Background current to beam. Background density of 10^5 (red), 10^7 (blue), 10^9 (green), and 10^{11} (purple) m^{-3} .

To begin to examine the evaporative cooling effect, we begin by integrating the three-dimensional electron distribution function to look at the flux out. We take one dimension to be "out" and allow the electrons with a minimum velocity of v_{min} or above to escape. We find that the flux out is

$$\frac{\exp\left[-\frac{m_e v_{\min}^2}{2kT}\right]n}{\sqrt{2\pi m_e/kT}}. \quad (14)$$

We define v_{\min} as

$$\sqrt{\frac{2kT(\delta f)^2}{m_e}}, \quad (15)$$

where δf is the ratio from well temperature to well depth that we would expect. Defining the ambient electron flux into the well as a more conventional $nv_{\text{avg}}/4$, we can set the two fluxes equal and solve for δf as

$$\delta f = \sqrt{-\ln\left[\frac{n_{bg}}{n} \sqrt{\frac{T_{bg}}{T}}\right]}. \quad (16)$$

As the density and temperature will invariably be lower in the background than the beam, the argument of the log will be negative, giving a real number for δf . Plugging in numbers for solar wind parameters at earth for a background density and temperature of $n_{bg}=10^8 \text{ m}^{-3}$ and $T_{bg}=2 \text{ eV}$, with fairly standard plume parameters of $n=10^{15} \text{ m}^{-3}$ and $T=2 \text{ eV}$, we get a δf of 4.55. A plot of (16) for various ratios of n_{bg}/n is seen in Figure 6.

While none of these results match the value of roughly 7 seen by Wang et al.,¹⁴ the magnitude of the change seems to be in line with the observation. Further refinement of the model may be able to better approximate the difference of well depth and beam temperature by including other effects such as ion flux, plasma sheath, and beam spreading, or use a different starting point, such as energy flux. One obvious issue is that even low-energy electrons from outside the well would have sufficient energy to escape out the other side without a collisional scattering into the well. Another is that we still see some sort of coupling in a vacuum chamber without a background plasma.

One interesting question this brings up is that of neutralizer placement. It has been observed from as early as SERT II that a neutralizer can function over a fairly significant distance, allowing for cross-neutralization in the case of multiple thrusters. After some initial period of operation, it is possible that the neutralizer will not provide a significant number of the neutralizing electrons. This raises the possibility that the neutralizer could be removed to some other location on the spacecraft to lessen its vulnerability to CEX ions from the beam plume. The plume will, however, remain the best conductive path away from the spacecraft, so it would be unlikely that there would ever be truly separate current paths.

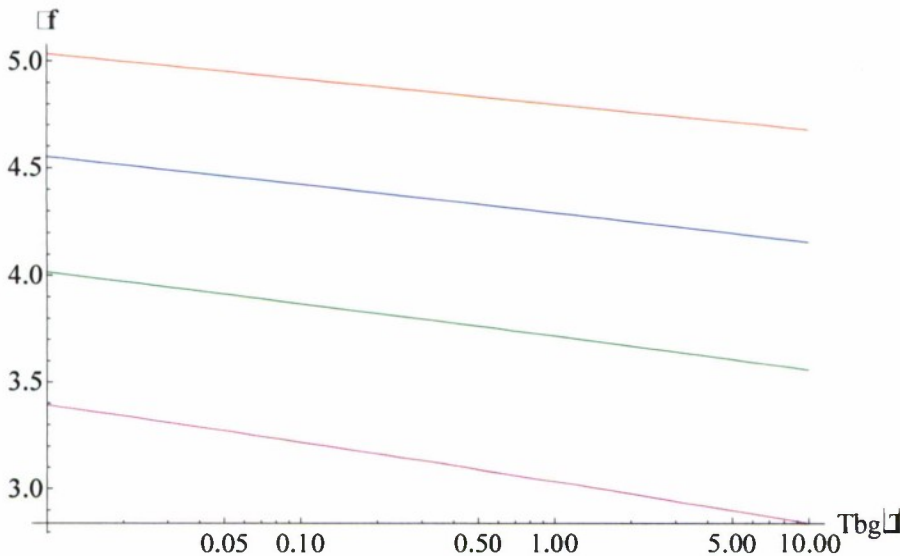


Figure 6: δf vs. T_{bg}/T for n_{bg}/n of 10^{-10} (red), 10^{-8} (blue), 10^{-6} (green), 10^{-4} (purple)

V. Conclusion

While not a complete generalized theory of ion beam neutralization, it appears that the formation of a potential well acts to trap electrons within the beam. Coulomb collisions are sufficient to deflect electrons into the beam in a manner that they do not have an exit vector even though they may be energetically capable of escaping. Once in the beam, further Coulomb collisions and collective instabilities can be shown to provide the necessary

matching of bulk velocities.

The well does not have to trap all electrons as the most energetic ones escape to the ambient environment while the background supplies electrons to the beam through normal thermal flux. We have shown that a basic momentum flux model provides a first order approximation of a functional ion thruster. Further development could provide a tool to predict thruster plume temperatures and energy flux to the background.

Future experimental work should determine if a potential well is deeper when background plasma is not present as suggested by the evaporation model. The collision and instability descriptions also do not require current coupling on their own. As this is a necessary condition for proper neutralizer function and observed in practice, it must be accounted for in any final theory.

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